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# Specific Gravity, Moisture Content, and Density Relationship for Wood

William T. Simpson



## Abstract

This report reviews the basis for determining values for the density of wood as it depends on moisture content and specific gravity. The data are presented in several ways to meet the needs of a variety of users.

Keywords: Specific gravity, density, wood weight

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Introduction $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $1$
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# Errata

#### Page 1, column 2, Equation (2)

 $D = (W_{\rm d}/V_{\rm g}) (1 + M/100)$ (2)

The term  $W_d/V_g$  is related to basic specific gravity  $G_b$  by

### Pages 9-12, Tables 5-8. Note on tables should read

Exceeds maximum possible moisture content

### Page 13, Appendix, Equation (la)

$$G_{\mathbf{b}} = (W_{\mathbf{d}}/V_{\mathbf{g}})p_{\mathbf{w}} \tag{1a}$$

where  $W_d$  is oven-dry weight of wood,  $V_{gg}$  is volume of green wood, and  $p_w$  is density of water.

### Page 13, Appendix, Equation (4a)

or

$$S = (V_g - V_M)/V_g$$

$$V_M = V_g (1 - S)$$
(4a)

# Specific Gravity, Moisture Content, and Density Relationship for Wood

William T. Simpson, Research Forest Products Technologist Forest Products Laboratory, Madison, Wisconsin

### Introduction

Knowledge about the density of wood is useful for estimating shipping weights. The density of wood depends on specific gravity and moisture content. A common way to present density data is in tabular and graphical form, where density in pounds per cubic foot or kilograms per cubic meter is shown for a series of specific gravity and moisture content values. This kind of system is shown in the *Wood Handbook* (FPL 1987). However, the system has several shortcomings.

One shortcoming is that density values in the *Wood Handbook* are shown for moisture contents above the maximum possible for those specific gravity values (Tables 1 and 2). Another shortcoming is that this system requires two steps: first, a graphical conversion of specific gravity from one volume base to another, and then a reading from a table. Furthermore, no equations are offered for direct calculation of density as a function of moisture content and specific gravity, which precludes easy use of the system in computer programs or spreadsheet calculations. Finally, the system is subject to misinterpretation and thus can yield erroneous density values if the user is not aware of the various definitions of specific gravity as it applies to wood.

This report discusses these shortcomings and develops equations, graphs, and tables for determining density as a function of moisture content and several common definitions of specific gravity.

# System for Determining Wood Density

The common definition of wood moisture content on a dry basis can be written as

$$W_{\rm g} = W_{\rm d}(1 + M/100)$$
 (1)

where  $W_{\rm g}$  is the green weight of wood (pounds or kilograms) at moisture content *M* (percent), and  $W_{\rm d}$  is the oven-dry weight of wood.

Dividing both sides of Equation (1) by the volume of wood at moisture content  $M(V_g)$ , the density *D* is

$$D = (W_{\rm d}/a_{\rm g})(1 + M/100)$$
(2)

The term  $W_{\rm d} V_{\rm g}$  is related to basic specific gravity  $G_{\rm b}$  by

$$W_{\rm d}/V_{\rm g} = p_{\rm w}G_{\rm b} \tag{3}$$

where  $p_w$  is the density of water (62.4 lb/ft<sup>3</sup> or 1,000 kg/m<sup>3</sup>) based on volume when above 30 percent moisture content. Substituting Equation (3) into Equation (2) results in the following equation for calculating density:

$$D = p_{\rm w}G_{\rm b}(1 + M/100) \tag{4}$$

A possible misapplication of Equation (4) is to use it at <30 percent moisture content with no correction in specific gravity for volumetric shrinkage. It is also possible to overlook or misunderstand the definition of specific gravity in the **Wood Handbook**, which could cause a user to read incorrect density values from the tables. Specific gravity is defined on the basis of volume at the tabulated moisture content- not on the more common basis of green volume, as is used to define basic specific gravity. Misunderstandings can cause errors 115 percent, as will be illustrated in another section of this report.

The *Wood Handbook* describes a graphical way to convert basic specific gravity to specific gravity based



Figure 1—Relation of specific gravity to moisture content. Follow line for specific gravity (volume when green) to desired moisture content, then read specific gravity (volume at current moisture content) from vertical axis. Source: Wood Handbook, Figures 3 and 4 (FPL 1987).

on volume at any of the tabulated moisture contents (Fig. 1). Density values can then be read from the tables after the conversion. Although this system of determining density values is valid, it requires two steps and the use of both a graph and a table. The same density information can be presented in one table or one graph.

The maximum moisture content of wood is reached when the cell walls and cell lumens are completely filled with water. When specific gravity is high, lumen volume is low and maximum moisture content is



Figure 2—Theoretical maximum possible moisture content of wood ( $M_{m \ a \ x}$ ) as a function of basic specific gravity ( $G_b$ ).

therefore restricted. This relationship can be estimated by

$$M_{\rm max} = (100/G_{\rm b})(1 - G_{\rm b}/G_{\rm w})$$
 (5)

where  $M_{\text{m a x}}$  is maximum moisture content and  $G_{\text{w}}$  is the specific gravity of wood substance, equal to approximately 1.54 (Skaar 1988). Equation (5) is shown in Figure 2.

Therefore, some density values at high specific gravity and moisture content values in Table 1 are not possible because the tabulated moisture content exceeds the maximum possible moisture content. These conditions will be discussed later in this report.

### **Density Tables**

### **Basic Specific Gravity**

Equation (4) can be used to change the basis of tabulated density values below 30 percent moisture content from specific gravity based on volume at tabulated moisture content to basic specific gravity if volume is corrected for shrinkage. Assuming a linear relationship between shrinkage and moisture content from 30 to 0 percent (Stamm 1964), the volumetric shrinkage *S* at any moisture content is

$$S = aS_{t} \tag{6}$$

where *a* is (30 - M)/30 and *S*<sub>t</sub> is total volumetric shrinkage from 30 to 0 percent moisture content.

Introducing Equation (6) into Equation (4) to account for shrinkage

$$D = p_{\rm w} G_{\rm b} \frac{(1 + M/100)}{1 - S} \tag{7}$$

Volumetric shrinkage data are necessary for using Equation (7). Each species has its own  $S_t$  value, and ideally there would be a density table for each species. However, this would be somewhat cumbersome and may not be necessary, given the inherent variability of wood and the known relationship between volumetric shrinkage and basic specific gravity. Stamm (1964) summarized the background on this relationship. The result is that volumetric shrinkage of both hardwoods and softwoods can be reasonably approximated by

$$S_{\rm t} = 0.265 G_{\rm b}$$
 (8)

Therefore, even if a density table is not constructed for each species, Equation (8) accounts for the major source of variation in shrinkage between species.

Combining Equations (6) to (8) results in

$$D = \frac{G_{\rm b}(1 + M/100)}{1 - 0.265 a G_{\rm b}} \tag{9}$$

The results of Equation (9) are listed in Table 3 and shown in Figure 3. Table 3 also shows maximum moisture content and density values. (The data in Table 3 are expressed in SI units in Table 4.)

The following example illustrates how using the incorrect specific gravity results in an incorrect density value. Suppose the user wants to know the density of wood when specific gravity is 0.56 and moisture content is 8 percent. If the volume basis for this given specific gravity is not clear, or if the user is not aware of the importance of the volume basis, the user might go directly to the tabular data (Table 1) and read a density of 37.7 lb/ft<sup>3</sup> (605 kg/m<sup>3</sup>). If the given specific gravity of 0.56 is the basic specific gravity, the correct density is 42.3 lb/ft<sup>3</sup> (679 kg/m<sup>3</sup>) (Table 3) and the incorrect value is in error by 11 percent. Although this may seem an unlikely error for specialists in wood technology, it does not seem unlikely for someone who does not realize the necessity of knowing the exact definition of specific gravity for specific tables or handbooks. These definitions, if present, are often cited in footnotes that can be overlooked or ignored.



Figure 3—Density of wood as a function of moisture content and basic specific gravity ( $G_b$ ); (a) English units, (b) SI units.

### **Other Specific Gravity Values**

Some references list specific gravity values on a basis other than green volume; in those cases, Table 3 is of no use in determining density values. Oven-dry volume (Hildebrand 1970) and volume at 12 percent moisture content (Chichignoud and others 1990) are common alternative bases. Equation (9) can be generalized for specific gravity baaed on volume at any moisture content if we develop the relationship between basic specific gravity and specific gravity based on the other volumes (Appendix). The result of this relationship is

$$G_{\rm b} = \frac{G_M}{1 + 0.265 a G_M} \tag{10}$$

where  $G_M$  is specific gravity based on volume at moisture content M. Equation (10) is substituted into Equation (9) for moisture contents <30 percent and into Equations (3) and (4) for moisture contents  $\geq$ 30 percent.

Table 5 is a density table for specific gravity based on volume at 12 percent moisture content ( $G_M = G_{1,2}$ )

(see Table 6 for SI units). Table 7 shows specific gravity values based on oven-dry volume ( $G_M = G_0$ ) (see Table 8 for SI values).

## **Concluding Remarks**

The validity and efficiency of a method for determining the density of wood at various combinations of specific gravity and moisture content depend on an understanding of the definition of specific gravity. Because specific gravity depends on wood volume, shrinkage must be accounted for when determining specific gravity below the fiber saturation point. The equations and tables described in this report can be used to determine density at any moisture content and specific gravity.

## References

**Chichiguoud, M.; Deon, G.; Detienne, P.; Parant, B.; and Vantomme, P.** 1990. Tropical timber atlas of Latin America. Yokohama, Japan: International Tropical Timber Organization, and Nogent-Sur-Marne Cedex, France: Centre Technique Forestier Tropical.

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Hildebrand, R. 1970. Kiln drying of sawn timber. Germany: Maschinenbau GmbH. 198 p.

Skaar, C. 1988. Wood-water relations. New York: Springer-Verlag. 283 p.

**Stamm, A.J.** 1964. Wood and cellulose science. New York: The Ronald Press. 549 p.

Table 1-Wood Handbook data for determining wood density<sup>a</sup>

	0.70	43.7	45.4	47.2	48.9	50.7	52.4	54.2	55.9	57.7	59.4	61.2	62.9	64.6	66.4	68.1	69.9	71.6	73.4	75.1	76.9	78.6	80.4	82.1	83.9	85.6	87.4	91.7	96.1	100.5	104.8	109.2	
	0.68	42.4	44.1	45.8	47.5	49.2	50.9	52.6	54.3	56.0	57.7	59.4	61.1	62.8	64.5	66.2	67.9	69.69	71.3	73.0	74.7	76.4	78.1	79.8	81.5	83.2	84.9	89.1	93.4	97.6	101.8	106.1	
	0.66	41.2	42.8	44.5	46.1	47.8	49.4	51.1	52.7	54.4	56.0	57.7	59.3	61.0	62.6	64.2	65.9	67.5	69.2	70.8	72.5	74.1	75.8	77.4	79.1	80.7	82.4	86.5	90.6	94.7	98.8	103.0	
:	0.64	39.9	41.5	43.1	44.7	46.3	47.9	49.5	51.1	52.7	54.3	55.9	57.5	59.1	60.7	62.3	63.9	65.5	67.1	68.7	70.3	71.9	73.5	75.1	76.7	78.3	79.9	83.9	87.9	91.9	95.8	<b>99.8</b>	
	0.62	38.7	40.2	41.8	43.3	44.9	46.4	48.0	49.5	51.1	52.6	54.2	55.7	57.3	58.8	60.4	61.9	63.4	65.0	66.5	68.1	69.6	71.2	72.7	74.3	75.8	77.4	81.2	85.1	89.0	92.9	96.7	
	0.60	37.4	38.9	40.4	41.9	43.4	44.9	46.4	47.9	49.4	50.9	52.4	53.9	55.4	56.9	58.4	59.9	61.4	62.9	64.4	65.9	67.4	68.9	70.4	71.9	73.4	74.9	78.6	82.4	86.1	89.9	93.6	
	0.58	36.2	37.6	39.1	40.5	42.0	43.4	44.9	46.3	47.8	49.2	50.7	52.1	53.6	55.0	56.5	57.9	59.4	60.8	62.3	63.7	65.1	66.6	68.0	69.5	70.9	72.4	76.0	79.6	83.2	86.9	90.5	
vityc	0.56	34.9	36.3	37.7	39.1	40.5	41.9	43.3	44.7	46.1	47.5	48.9	50.3	51.7	53.1	54.5	55.9	57.3	58.7	60.1	61.5	62.9	64.3	65.7	67.1	68.5	69.9	73.4	76.9	80.4	83.9	87.4	
ocific grav	0.54	33.7	35.0	36.4	37.7	39.1	40.4	41.8	43.1	44.5	45.8	47.2	48.5	49.9	51.2	52.6	53.9	55.3	56.6	58.0	59.3	60.7	62.0	63.3	64.7	66.0	67.4	70.8	74.1	7.5	80.9	84.2	
ls of spe	0.52	32.4	33.7	35.0	36.3	37.6	38.9	40.2	41.5	42.8	<b>4</b> 1.4	45.4	46.7	48.0	49.3	50.6	51.9	53.2	54.5	55.8	57.1	58.4	59.7	61.0	62.3	63.6	64.9	68.1	7.4	74.6	6.17	81.1	
ious leve	0.50	31.2	32.4	33.7	34.9	36.2	37.4	38.7	39.9	41,2	42.4	43.7	44.9	46.2	47.4	48.7	49.9	51.2	52.4	53.7	54.9	56.2	57.4	58.7	59.9	61.2	62.4	65.5	68.6	71.8	74.9	78.0	
3) at var	0.48	30.0	31.2	32.3	33.5	34.7	35.9	37.1	38.3	39.5	40.7	41.9	43.1	44.3	45.5	46.7	47.9	49.1	50.3	51.5	52.7	53.9	55.1	56.3	57.5	58.7	59.9	62.9	62.9	68.9	71.9	74.9	
sity (Ib/ft	0.46	28.7	29.9	31.0	32.1	33.3	34.4	35.6	36.7	37.9	39.0	<del>4</del> 0.2	41.3	42.5	43.6	44.8	45.9	47.1	<b>48.2</b>	49.4	50.5	51.7	52.8	54.0	55.1	56.3	57.4	60.3	63.1	66.0	68.9	71.8	
Den	0.44	27.5	28.6	29.7	30.8	31.8	32.9	34.0	35.1	36.2	37.3	38.4	39.5	40.6	41.7	42.8	43.9	45.0	46.1	47.2	48.3	49.4	50.5	51.6	52.7	53.8	54.9	57.7	60.4	63.1	65.9	68.6	
	0.42	26.2	27.3	28.3	29.4	30.4	31.4	32.5	33.5	34.6	35.6	36.7	37.7	38.8	39.8	40.9	41.9	43.0	44.0	45.1	46.1	47.2	48.2	49.3	50.3	51.4	52.4	55.0	57.7	60.3	62.9	65.5	
	0.40	25.0	26.0	27.0	28.0	29.0	30.0	31.0	31.9	32.9	33.9	34.9	35.9	36.9	37.9	38.9	39.9	40.9	41.9	42.9	43.9	44.9	45.9	46.9	47.9	48.9	49.9	52.4	54.9	57.4	59.9	62.4	
	0.38	23.7	24.7	25.6	26.6	27.5	28.5	29.4	30.4	31.3	32.2	33.2	34.1	35.1	36.0	37.0	37.9	38.9	39.8	40.8	41.7	42.7	43.6	44.6	45.5	46.5	47.4	49.8	52.2	54.5	56.9	59.3	
	0.36	22.5	23.4	24.3	25.2	26.1	27.0	27.9	28.8	29.7	30.6	31.4	32.3	33.2	34.1	35.0	35.9	36.8	37.7	38.6	39.5	<b>4</b> 0.4	41.3	42.2	<b>43.</b> 1	<b>4</b> 0	4.9	47.2	49.4	51.7	53.9	56.2	
	0.34	21.2	22.1	22.9	23.8	24.6	25.5	26.3	27.2	28.0	28.9	29.7	30.6	31.4	32.2	33.1	33.9	34.8	35.6	36.5	37.3	38.2	39.0	39.9	40.7	41.6	42.4	44.6	46.7	48.8	50.9	53.0	
	0.32	20.0	20.8	21.6	22.4	23.2	24.0	24.8	25.6	26.4	27.2	28.0	28.8	29.6	30.4	31.2	31.9	32.7	33.5	34.3	35.1	35.9	36.7	37.5	38.3	39.1	39.9	41.9	43.9	45.9	47.9	49.9	
	0.30	18.7	19.5	20.2	21.0	21.7	22.5	23.2	24.0	24.7	25.5	26.2	27.0	27.7	28.5	29.2	30.0	30.7	31.4	32.2	32.9	33.7	34.4	35.2	35.9	36.7	37.4	39.3	41.2	43.1	44.9	46.8	
qΜ	(%)	0	4	œ	12	16	20	24	28	32	36	4	4	48	52	56	60	64	68	72	76	80	84	88	92	96	5 5	110	120	130	40	150	

<sup>a</sup>Values based on mass when ovendry and volume at tabulated moisture content. From the *Wood Handbook* (FPL 1987); Tables 3 to 7. <sup>b</sup>M is moisture content. <sup>c</sup>D = 62.4Gb(1 + M/100) for all levels of moisture content.

Table 2—Wood Handbook data in Si units of measurement M

	0.70	200	728	756	784	812	840	868	896	924	952	086	1008	1036	1064	1092	1120	1148	1176	1204	1232	1260	1288	1316	1344	1372	1400	1470	1540	1610	1680	1750
	0.68	680	707	734	762	789	816	843	870	868	925	952	979	1006	1034	1061	1088	1115	1142	1170	1197	1224	1251	1278	1306	1333	1360	1428	1496	1564	1632	120
	0.66	660	686	713	739	766	792	818	845	871	868	924	950	977	1003	1030	1056	1082	1109	1135	1162	1188	1214	1241	1267	1294	1320	1386	1452	1518	1584	1650
-	0.64	640	666	691	717	742	768	794	819	845	870	896	922	947	973	966	1024	1050	1075	1101	1126	1152	1178	1203	1229	1254	1280	1344	1408	1472	1536	1600
	0.62	620	645	670	694	719	744	769	794	818	843	868	893	918	942	967	992	1017	1042	1066	1091	1116	1141	1166	1190	1215	1240	1302	1364	1426	1488	1550
	0.60	600	624	648	672	969	720	744	768	792	816	840	864	888	912	936	960	984	1008	1032	1056	1080	1104	1128	1152	1176	1200	1260	1320	1380	1440	1500
	0.58	580	603	626	650	673	<b>696</b>	719	742	766	789	812	835	858	882	905	928	951	974	<b>866</b>	1021	1044	1067	1090	1114	1137	1160	1218	1276	1334	1392	1450
vitya	0.56	560	582	605	627	650	672	694	717	739	762	784	806	829	851	874	896	918	941	963	986	1008	1030	1053	1075	1098	1120	1176	1232	1288	1344	<b>4</b>
ific gra	0.54	540	562	583	605	626	648	670	691	713	734	756	778	799	821	842	864	886	907	929	950	972	994	1015	1037	1058	1080	1134	1188	1242	1296	1350
is of spe	0.52	520	541	562	582	603	624	645	666	686	707	728	749	70	290	811	832	853	874	894	915	936	957	978	<b>866</b>	1019	1040	1092	1144	1196	1248	1300
ous leve	0.50	500	520	540	560	580	600	620	640	660	680	20	720	740	760	780	800	820	840	860	8B0	006	920	940	960	980	80	1050	1100	1150	1200	1250
· .×	· · ·																											•				
3) at vario	0.48	480	499	518	538	557	576	595	614	634	653	672	691	710	730	749	768	787	806	826	845	864	883	902	922	941	960	1008	1056	1104	1152	1200
ity (kg/m <sup>3</sup> ) at vari	0.46 0.48	460 480	478 499	497 518	515 538	534 557	552 576	570 595	589 614	607 634	626 653	644 672	662 691	681 710	699 730	718 749	736 768	754 787	773 806	791 826	810 845	828 864	846 883	865 902	883 922	902 941	920 960	966 1008	1012 1056	1058 1104	1104 1152	1150 1200
Density (kg/m <sup>3</sup> ) at vari	0.44 0.46 0.48	440 460 480	458 478 499	475 497 518	493 515 538	510 534 557	528 552 576	546 570 595	563 589 614	581 607 634	598 626 653	616 644 672	634 662 691	651 681 710	669 699 730	686 718 749	704 736 768	722 754 787	739 773 806	757 791 826	774 810 845	792 828 864	810 846 883	827 865 902	845 883 922	862 902 941	880 920 960	924 966 1008	968 1012 1056	1012 1058 1104	1056 1104 1152	1100 1150 1200
Density (kg/m <sup>3</sup> ) at vari	0.42 0.44 0.46 0.48	420 440 460 480	437 458 478 499	454 475 497 518	470 493 515 538	487 510 534 557	504 528 552 576	521 546 570 595	538 563 589 614	554 581 607 634	571 598 626 653	588 616 644 672	605 634 662 691	622 651 681 710	638 669 699 730	655 686 718 749	672 704 736 768	689 722 754 787	706 739 773 806	722 757 791 826	739 774 810 845	756 792 828 864	773 810 846 883	790 827 865 902	806 845 883 922	823 862 902 941	840 880 920 960	832 924 966 1008	924 968 1012 1056	966 1012 1058 1104	1008 1056 1104 1152	1050 1100 1150 1200
Density (kg/m <sup>3</sup> ) at vari	0.40 0.42 0.44 0.46 0.48	400 420 440 460 480	416 437 458 478 499	432 454 475 497 518	448 470 493 515 538	464 487 510 534 557	480 504 528 552 576	496 521 546 570 595	512 538 563 589 614	528 554 581 607 634	544 571 598 626 653	560 588 616 644 672	576 605 634 662 691	592 622 651 681 710	608 638 669 699 730	624 655 686 718 749	640 672 704 736 768	656 689 722 754 787	672 706 739 773 806	688 722 757 791 826	704 739 774 810 845	720 756 792 828 864	736 773 810 846 883	752 790 827 865 902	768 806 845 883 922	784 823 862 902 941	800 840 880 920 960	840 832 924 966 1008	880 924 968 1012 1056	920 966 1012 1058 1104	960 1008 1056 1104 1152	1000 1050 1100 1150 1200
Density (kg/m <sup>3</sup> ) at vari	0.38 0.40 0.42 0.44 0.46 0.48	380 400 420 440 460 480	395 416 437 458 478 499	410 432 454 475 497 518	426 448 470 493 515 538	441 464 487 510 534 557	456 480 504 528 552 576	471 496 521 546 570 595	486 512 538 563 589 614	502 528 554 581 607 634	517 544 571 598 626 653	532 560 588 616 644 672	547 576 605 634 662 691	562 592 622 651 681 710	578 608 638 669 699 730	593 624 655 686 718 749	608 640 672 704 736 768	623 656 689 722 754 787	638 672 706 739 773 806	854 688 722 757 791 826	669 704 739 774 810 845	684 720 756 792 828 864	699 736 773 810 846 883	714 752 790 827 865 902	730 768 806 845 883 922	745 784 823 862 902 941	760 800 840 880 920 960	798 840 832 924 966 1008	836 880 924 968 1012 1056	874 920 966 1012 1058 1104	912 960 1008 1056 1104 1152	950 1000 1050 1100 1150 1200
Density (kg/m <sup>3</sup> ) at vari	0.36 0.38 0.40 0.42 0.44 0.46 0.48	360 380 400 420 440 460 480	374 395 416 437 458 478 499	389 410 432 454 475 497 518	403 426 448 470 493 515 538	418 441 464 487 510 534 557	432 456 480 504 528 552 576	446 471 496 521 546 570 595	461 486 512 538 563 589 614	475 502 528 554 581 607 634	490 517 544 571 598 626 653	504 532 560 588 616 644 672	518 547 576 605 634 662 691	533 562 592 622 651 681 710	547 578 608 638 669 699 730	562 593 624 655 686 718 749	576 608 640 672 704 736 768	590 623 656 689 722 754 787	605 638 672 706 739 773 806	619 854 688 722 757 791 826	634 669 704 739 774 810 845	648 684 720 756 792 828 864	<b>662 699 736 773 810 846 883</b>	677 714 752 790 827 865 902	691 730 768 806 845 883 922	706 745 784 823 862 902 941	720 760 800 840 880 920 960	756 798 840 832 924 966 1008	792 836 880 924 968 1012 1056	828 874 920 966 1012 1058 1104	864 912 960 1008 1056 1104 1152	900 950 1000 1050 1100 1150 1200
Density (kg/m <sup>3</sup> ) at vari	0.34 0.36 0.38 0.40 0.42 0.44 0.46 0.48	340 360 380 400 420 440 460 480	354 374 395 416 437 458 478 499	367 389 410 432 454 475 497 518	381 403 426 448 470 493 515 538	394 418 441 464 487 510 534 557	408 432 456 480 504 528 552 576	422 446 471 498 521 548 570 595	435 461 486 512 538 563 589 614	449 475 502 528 554 581 607 634	482 490 517 544 571 598 626 653	476 504 532 560 588 616 644 672	490 518 547 576 605 634 662 691	503 533 562 592 622 651 681 710	517 547 578 608 638 669 699 730	530 562 593 624 655 686 718 749	544 576 608 640 672 704 736 768	558 590 623 656 689 722 754 787	571 605 638 672 706 739 773 806	585 619 854 688 722 757 791 826	598 634 669 704 739 774 810 845	612 648 684 720 756 792 828 864	626 662 699 736 773 810 846 883	639 677 714 752 790 827 865 902	<b>5</b> 53 591 730 768 806 845 883 922	<b>666</b> 706 745 784 823 862 902 941	680 720 760 800 840 880 920 960	714 756 798 840 832 924 966 1008	748 792 836 880 924 968 1012 1056	782 828 874 920 966 1012 1058 1104	816 864 912 960 1008 1056 1104 1152	850 900 950 1000 1050 1100 1150 1200
Density (kg/m <sup>3</sup> ) at vari	0.32 0.34 0.36 0.38 0.40 0.42 0.44 0.46 0.48	320 340 360 380 400 420 440 460 480	333 354 374 395 416 437 458 478 499	346 367 389 410 432 454 475 497 518	358 381 403 426 448 470 493 515 538	371 394 418 441 464 487 510 534 557	384 408 432 456 480 504 528 552 576	397 422 446 471 496 521 546 570 595	410 435 461 486 512 538 563 589 614	422 449 475 502 528 554 581 607 634	435 482 490 517 544 571 598 626 653	448 476 504 532 560 588 616 644 672	461 490 518 547 576 605 634 662 691	474 503 533 562 592 622 651 681 710	486 517 547 578 608 638 669 699 730	499 530 562 593 624 655 686 718 749	512 544 576 608 640 672 704 736 768	525 558 590 623 656 689 722 754 787	538 571 605 638 672 706 739 773 806	550 585 619 854 688 722 757 791 826	563 598 634 669 704 739 774 810 845	576 612 648 684 720 756 792 828 864	589 626 662 699 736 773 810 846 883	602 639 677 714 752 790 827 865 902		627 666 706 745 784 823 862 902 941	640 680 720 760 800 840 880 920 960	672 714 756 798 840 832 924 966 1008 ·	704 748 792 836 880 924 968 1012 1056	736 782 828 874 920 966 1012 1058 1104	768 816 864 912 960 1008 1056 1104 1152	800 850 900 950 1000 1050 1100 1150 1200
Density (kg/m <sup>3</sup> ) at vari	0.30 0.32 0.34 0.36 0.38 0.40 0.42 0.44 0.46 0.48	300 320 340 360 380 400 420 440 460 480	312 333 354 374 395 416 437 458 478 499	324 346 367 389 410 432 454 475 497 518	336 358 381 403 426 448 470 493 515 538	348 371 394 418 441 464 487 510 534 557	360 384 408 432 456 480 504 528 552 576	372 397 422 446 471 496 521 546 570 595	384 410 435 461 486 512 538 563 589 614	396 422 449 475 502 528 554 581 607 634	408 435 482 490 517 544 571 598 626 653	420 448 476 504 532 560 588 616 644 672	432 461 490 518 547 576 605 634 682 691	444 474 503 533 562 592 622 651 681 710	456 486 517 547 578 608 638 669 699 730	468 499 530 562 593 624 655 686 718 749	480 512 544 576 608 640 672 704 736 768	492 525 558 590 623 656 689 722 754 787	504 538 571 605 638 672 706 739 773 806	516 550 585 619 854 688 722 757 791 826	528 563 598 634 669 704 739 774 810 845	540 576 612 648 684 720 756 792 828 864		564 602 639 677 714 752 790 827 865 902		588 627 666 706 745 784 823 862 902 941	600 640 680 720 760 800 840 880 920 960	630 672 714 756 798 840 832 924 966 1008 ·	660 704 748 792 836 880 924 968 1012 1056 ·	690 736 782 828 874 920 966 1012 1058 1104	720 768 816 864 912 960 1008 1056 1104 1152	

 $^{a}D = 1000G_{b}(1 + M/100)$  for all levels of moisture content.

	0,70	ਲ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ਸ਼ ਲ਼੶੶੶੶੶੶੶੶੶੶੶੶੶੶੶੶੶	82 77.77	
	0.68	222328282828282828282828282828282828282	82 7.3	
	0.66	88 89 80 80 80 80 80 80 80 80 80 80	87 78.8	
	0.64	######################################	91 78.4	
	0.62	# 888.0 7,27,72888890988890 7,27,727888890988890 7,27,7279888909797 7,27,727988890979 7,27,727988890979 7,27,7279 7,27,720 7,27,720 7,27,7	96 75.9	
	0.60	44. 45. 45. 45. 45. 45. 45. 45. 45. 45.	102 75.5	
volume) <sup>8</sup>	0.58	84444444444444444444444444444444444444	107 75.1	
ht/green	0.56	Hin bo	114 74.6	
ndry weig	0.54	88444444444444444444444444444444444444	120 74.2	+ M/100).
Bivity (ove	0.52	여행 888 889 99 99 99 99 99 99 99 99 99 99 9	127 73.8	2.4Gb(1 +
specific gr	0.50	8886888884494444488688688688686 000000000400040004000400040088	135 73.3	ent, D = 6
levels of s	0,48	¥%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%	143 72.9	> 30 pero
t various	0.46	ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਫ਼ਖ਼ <i>ਖ਼ਖ਼ਖ਼ਖ਼ਖ਼ਖ਼ਫ਼</i> ਫ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼	152 72.4	O; for M >
(DM3) a	0.44	ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਫ਼ਖ਼ਖ਼ਖ਼ਖ਼ਖ਼ਖ਼ਫ਼ਖ਼ਫ਼ਫ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਲ਼ਫ਼ਫ਼ਫ਼ ੶੶੶੶੶੶੶੶੶੶	162 720	enn - Oe
Density	0.42	ୢୄ ଽ୵ଌ୶୵ୄ୵୵୵ଌଌୡଽୡୡୡୡୡୢଌୢଌୢଌୢଌୢୄଌୡୢ ଌ୵ଌ୶୵୵୵୵ଌଌୡଽୡ୶ୡୡୡୡୢଌୢଌୢଌୢୢୢୢ୷ୠୢୢୄ୷ୠୢୢୄଌୢୢଌୢୄ <mark>ଌ</mark>	57- 8.K 8.K	) = e e.e.
	040	ୄ୵ୡୄୄୄୄୄୄୄଝୄୄୄୄୄୄୄୄୄୄୄୄୄୄ ଽୡୄୄୄୄୄୄୡୡୄଌୄଌଽୄଌୄୡୢୄୡୢୄୡୢୄୡୢୄୡୢୡୢୄୡୢୄୡୢୄୡୄୢୡୄୢୡୄୢୡୢୄୡୢୄୡ	185 71.1	SaGb), w
	88.0	<i>₭₭₭₭₭₭₭₭₭₭₭₭₭₭₦₦₦₦₦₦₦₦₦₦₦₦₦₦₦₦₦₦₦₦₦₦₦</i>	7.98 7.7	((1 - 0.26)
	980	<i>ұқққұрққққсққққққққққ</i> <b>8844844444</b> ®л-Гойтогоссисторасторасторастора 8.1-Гойтоссисторасторасторастора	213 213	(001/W +
	0.34	ୄୄୄୄୄୄୄୄୄୄ ଝୄ୶ୄୄୄୄୄୄୄୄୄୄୄୄୄ ଝୄ୶ୢୄୄୄୄୄୄୄୄୄୄ	888 888	32.4Gb(1
	0.32	<i>୰୳ୠୠୡୡୡୡଌଌଌଌଌଌଌଌୡୡୡୡୡୡ</i> ଌଌଌୡୡୡୡୡୡଌଌଌୡୡୡୡୡୡଌଌଌୡୡଌଌ ଌ୕ଽ୕ୖ୶୷୷୷୷୶୰୶୶ୡଽ୶୶୶୵୶୷୷୶୷୶୷୶୶୶୶୶୶୶୶୶	248 69.4	ant, D = 6
	0:30	88227788228882288882888888888888884 <b>444488</b> 8828 20220828288888888888888888888888	880 89 90 90 90 90 90 90 90 90 90 90 90 90 90	< 30 pero
X	(%)	o4∞5≈8228888448888888888888885585858588888888	Max M Max D Max D	afor M

Table 3—Results of Equation (9) for determining wood density

	0.70	855 865 865 888 888 888 888 888 888 888	78 1245	
	0.68	829 854 854 8554 8861 8861 8875 9255 972 972 972 972 972 972 972 972 972 972	82 1238	
	0.66	800 800 818 825 825 825 825 825 825 825 1110 825 1110 825 1110 825 1110 825 1110 825 1110 825 1110 825 1110 825 827 827 827 827 827 827 827 827 827 827	87 1231	
	0.64	778882 8828 8828 8828 8829 8829 8829 882	91 1224	
	0.62	755 761 770 787 787 788 888 888 888 888 888 888	96 1217	
	0.60	7.73           7.73           7.75	102 1210	
olume) <sup>a</sup>	0.58	688 696 696 696 776 775 775 775 775 775 775 775 775 77	107 1203	
Vgreen vo	0.56	658 6679 6679 6679 6679 6688 8887 7724 7726 7726 8898 9918 8829 9911 10055 11003 9911 10035 110035 110035 110035 110035 110035 9911 10055 9911 10055 99110 10055 99110 10055 99110 10055 90110000000000	114 1196	100).
dry weight	0.54	6.8 6.8 6.8 6.8 6.8 6.8 6.8 6.8 7 7 7 7 7 7 8 8 8 8 8 8 8 8 8 8 8 8 8	120 1189	/W + 1)qe
rity (oven	0.52	66 67 67 67 67 67 67 67 77 77 77 77 77 7	127 1182	= 10000
cific grav	0.50	555 557 558 558 558 558 558 558 558 558	135 175	eroent, D
els of spe	0.48	83888888888888888888888888888888888888	143 1168	M > 30 p
rious leve	0.46	555 555 555 556 556 556 556 556 556 556	152 1161	)/30; for
n <sup>3</sup> ) at va	0.44	553 553 553 553 553 553 553 553 553 553	162 1154	W - 06) -
nsity (kg/i	0.42	1382 1382 1382 1382 1382 1382 1382 1382	173 1147	where a -
Der	0.40	4488488822223486268828862222288886222228888666666 48884888225334862688388886222228888888666666	185 1140	65aGb),
	0.38	88888888888888888888888888888888888888	198 1133	)/(1 - 0.2
	0.36	884444444444685555568855855555555555555	213 1126	+ M/100
	0.34	2888 2888 2888 2888 2888 2888 2888 288	229 1119	000G <sub>b</sub> (1
	0.32	359 359 359 359 359 359 359 359 359 359	248 1112	nt, D = 1
	0:30	88888888888888888888888888888888888888	268 1105	30 perce
Z	(%)	o4∞59228288448388888628888886558588888888888888888888	Max M Max D	aFor M <

Table 4-Results of Equation (9) in SI units

	68 0.70	732728888829282888882929292929292929292929	98 94 3.8 76.2	
	0.66 0.	<b>444444444444444444444444444444444444</b>	102 75.4 75	
	0.64	88888888888888888888888888888888888888	107 75.1	
	0.62	44444444444444444444444444444444444444	112 74.7	
	0.60	64444644464666666666666666666666666666	118 74.4	
	0.58	88889444444444444444444444444444444444	123 74.0	
ravity <sup>a</sup>	0.56	88888944444444444444444444444444444444	129 73.6	
pecific g	2 0.54	C 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	3 136 3 73.3	
/els of sl	0.52	4%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%	144	
rious lev	0.50	%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%	151	ľ
3) at va	0.46	E888888888888844444444486288888888888888	156	
th (b/ft	1 0.46	882888888888888449848484888888888888888	168	
Den	0.4	828855888888558884494444484865588855888 811,81,91,91,91,90,90,00,00,00,00,00,00,00,00,00,00,00,	172	
	0.42	78888855555555555555555555555555555555	189	
	0.40	88288888888888888888888888888888888888	70.6	
	0.36	488887288888888888888888844444468888888888	237	
	1 0.36	82828882888888888888888888888888888888	8.69	
	0.34	772224242424242424242424242424242424242	8 245 69.4	
	0.32	822288888888888888888888888888888888888	4 263 69.1	
	0.30		587 58.7	
Σ	(%)	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	Max M Max C	

Table 5-Wood density based on volume at 12 percent moisture content

	0.70	7756 7756 7756 7756 8857 9577 9577 9577 9577 11159 111	94 1220	
	0.68	222 222 222 222 222 222 222 222 222 22	98 1215	
	0.66	710 757 757 757 757 755 755 755 755 755 75	102 1209	
	0.64	888 888 888 888 888 888 888 888 888 88	107 1203	
	0.62	664           674           674           674           674           674           674           674           674           674           704           705           674           705           706           713           713           713           713           713           713           7145           713           713           7145           7145           7103           926           926           926           926           927           926           926           926           926           926           926           926           926           926           926           926           926           926           927           928           928           928           928           928	112 1198	
	0.60	641 652 652 652 652 652 652 652 700 700 745 7767 7767 7767 7767 7767 7767 7767	118 1192	
	0.58	618 626 640 655 655 655 655 655 655 655 655 655 65	123 1186	
avity <sup>a</sup>	0.56	595           617         617           627         627           627         627           628         628           629         628           629         628           620         629           620         720           621         760           622         720           623         629           624         720           720         720	129 1180	
ecific gra	0.54	ss magd	136 1174	
els of sp	0.52	EX8 EX8 EX8 EX8 EX8 EX8 EX8 EX8	143 1168	
ous leve	0.50	550 550 550 550 550 550 550 550 550 550	151 1162	
) at vari	0.48	557 557 557 557 557 557 557 557 557 557	159 1156	
/ (kg/m <sup>3</sup>	0.46	484 505 505 505 505 505 505 505 505 505 50	168 1150	
Density	0.44	444 445 555 555 555 555 555 555 555 555	178 1144	
	0.42	440 450 450 450 450 450 450 450 450 450	189 1138	
	0.40	44888444464446444444444444444444444444	201 1132	
	0.38	868 868 868 868 868 868 868 868	214 1125	
	0.36	3344 3344 3344 3344 3355 3558 3558 3558	229 1119	
	0.34	88888844444444444444444444444444444444	245 1113	
	0.32	330 350 350 350 350 350 350 550 550 550	263 1107	
	0:30	3219 3219 3219 3219 3219 3219 3219 3219	284 1100	
¥	(%)	o4∞5555485884483883856888888555555555555555	Max M Max D	

**BFor** M < 30 percent,  $D = 1000G_b(1 + M/100)/(1 - 0.265 aG_b)$ , where a = (30 - M/30 and  $G_b = G_{12}(1 + 0.265aG_{12})$ ; for M > 30 percent,  $D = 1000G_b(1 + M/100)$ .

Table 6-SI values for data in Table 5

	0.70	84488844488825834868888888888888888888888888888888888	104 75.3	
	0.68	4554448484888666888866688886886666 4555448888888888	109 75.0	
	0.66	44488444888888888888888888888888888888	113 74.7	(1100).
	0.64	888444444444882838888888888888888888888	118 74.4	b(1 + M
	0.62	888959595959595959595959595959595959595	123 74.0	- 62.4G
	0.60	o sture Second Second Se Second Second Sec	128 73.7	ænt, D
	0.58	88.5.338.5.338.5.5 2.200.00.01.2.5.5.5.5.5.5.5.5.5.5.5.5.5.5.5.5.5.5.	134 73.4	30 perc
Ŋa	0.56	88888888888888848484848484888888888888	73.0	for M >
lic gravi	0.54	88888888888888888888888888888888888888	27	5aG0);
of speci	).52	ХХХХХХХХХХХХХХХХХХХХХХХХХХХХХХХХХХХХХ	2.4	+ 0.26
levels (	.50 (	222022428882888002224280022049200882 0000184004002000000000000000000000000000	62 2.0 7	о = G <sub>0</sub> (1
various	48	00000000000000444444444600000000000000	22	and G
offt 3) at	<b>8</b> 0	<u>, , , , , , , , , , , , , , , , , , , </u>	64	W/30
sity (Ib	ò	88888558888888888888888888888888888888	77	- 06)
Den	0. 4	78888888888888888888888888888888888888	189 71.0	lere a -
	0 4	88822888288888888888888888888888888888	200 201 200	Gb), wh
	0.40	<i><i>ჯჯფფევფფვფფფფფეფეფფფფფფფფფფფფფფ</i></i>	211 70.3	0.265 <i>a</i>
	0.38	8253320082525475747575757575757575757575757575757	225 69.9	- 1)/(00
	0.36	82828282828282888888888888888888888888	239 69.6	1 + M/1
	0.34	22222222222222222222222222222222222222	256 69.2	62.4Gb(
	0.32	88.877,977,977,977,977,977,977,977,977,977,	274 68.8	H, D =
	0.30	860-82828282828282828282828282828282828282	295 68.5	30 perce
N	(%)	o4∞5≈52%88%844%8%88%%2552%84%8558%88%	Max M Max D Max D	<sup>a</sup> For M < :

Table 7---Wood density based on ovendry volume

		700 7200 7	104	
	0.68	888 872 872 872 872 872 872 872 872 872	1202 1202	
	0.66	882 882 882 882 882 882 882 882	113 1197	
	0.64	882 882 882 882 882 882 882 882	118 1192	- M/100).
	0.62	620 621 622 623 623 623 623 623 623 623 623 623	123 1186	00Gb(1 4
	0.60	moistur	128 1181	r, D = 10
	0.58	888 888 888 888 888 888 888 888	134 1176	30 percei
- the second	0.56	551           552	140 171	for M >
enerifi	0.54	Excess Ex	147 1165	65aG <sub>0</sub> )
lavale o	0.52	222 222 222 222 222 222 222 222	154 1160	(1 + 0.2
various	0.50	5522 5511 5522 5522 5522 5522 5521 5522 5522 5522 5521 5522 5525 5555 5555 5555 5555 5555 5555 5555 5555	162 1155	3 <b>b =</b> G0
/m <sup>3</sup> / at	0.48	888 888 888 888 888 888 888 888 888 88	1149	0 and 0
ansity (kr	0.46	8448885558848868888888887788476888888888699 8448885558848868888888888875884768888888888	57 178	30 - M/3
	0.44	44444444444444444444444444444444444444	189 1138	) <b>-</b> a <b>-</b> (
	0.42	844 4 84 84 84 84 84 84 84 84 84 84 84 8	200 1132	iGb), wh
	0.40	6444688875888888888888888888888888888888	211 1127	- 0.265a
	0.38	88888888888888888888888888888888888888	225 1121	100)/(1
	0.36	88888888888888888888888888888888888888	239 1115	(1 + M
	0.34	346 346 346 346 346 346 346 346 346 346	256 1109	- 1000G
	0.32	3220 3220 3220 3220 3220 3220 3220 3220	274 1103	sent, D =
	0:30	88878888888888888888888888888888888888	295 1097	: 30 perc
Z	(%)	o4∞2≈8%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%	Max M Max D	aror M <

Table 8—Si values for data in Table 7

12

I

# Appendix. Relationship of Specific Gravity Values

In this appendix, we describe the general relationship between basic specific gravity and specific gravity values baaed on volume at other moisture content levels.

Basic specific gravity is defined as

$$G_{\rm b} = (W_{\rm d}/V_{\rm d})/p_{\rm w} \tag{1a}$$

where  $W_d$  is oven-dry weight of wood,  $V_d$  is volume of dry wood, and  $p_{w_w}$  is density of water.

Specific gravity baaed on volume at any other moisture content M is defined as

$$G_M = (W_d/V_M)/p_w \tag{2a}$$

Equating Equations (1a) and (2a) through  $W_d$ 

$$G_M = G_b V_g / V_M \tag{3a}$$

where  $V_{\rm g}$  is volume of green wood.

Volumetric shrinkage S from  $\geq$ 30 percent to <30 percent moisture content is

or

$$S = (V_{\rm g} - V_M)/V_M$$

$$V_M = V_g(1-S) \tag{4a}$$

Substituting Equation (4a) into Equation (3a)

$$G_{\mathbf{b}} = G_{\mathbf{M}}(1 - S) \tag{5a}$$

Assuming the linear shrinkage of Equation (6) in the text  $(S = aS_t)$ 

$$G_{\mathbf{b}} = G_{\boldsymbol{M}}(1 - aS_{\mathbf{t}}) \tag{6a}$$

where a is (30 - M)/30.

The next step is to express  $S_t$  in Equation (6a) in terms of  $G_M$ . Substituting Equation (6a) into Equation (8) ( $S_t = 0.265 G_b$ ) in the text

$$S_{\rm t} = 0.265 G_M (1 - aS_{\rm t}) \tag{7a}$$

Solving for  $S_t$ 

$$S_{t} = \frac{0.265G_{M}}{1 + 0.265aG_{M}} \tag{8a}$$

and substituting Equation (8a) into Equation (6a)

$$G_{\rm b} = \frac{G_M}{1 + 0.265 a G_M} \tag{9a}$$

which can be substituted into Equation (9) in the text to calculate density based on the volume at any moisture content. Equation (9a) can also be used to calculate the graphical relationship between moisture content, basic specific gravity, and specific gravity based on volume at any moisture content, as shown in Figure 1 of the text.

The value of M in the term a ((30 - M)/30) in Equation (9a) is the moisture content for the volume base of the specific gravity. The value of M in the term **a** in Equation (9) in the text is the moisture content at which the density is calculated (that is, 0 to 200 percent in Tables 5 and 7). If this latter value of M is taken as the moisture content of the specific gravity volume base, the resulting density values will be baaed on the specific gravity at the volume of the tabulated moisture content, that is, the same as that in the tables in the *Wood Handbook*. Note also in Equation (9a) that when M = 30, a = 0, and  $G_M =$  $G_{\rm b}$ , text Equation (9) reverts to text Equations (3) and (4).